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[redacted]

IS-LM-Model
A Dynamic Approach

Summary:

Following the well known terminology suggested by Hicks (1937), the intersection of the IS- and the LM-curve determines the point in which real and monetary equilibrium are obtained simultaneously. Allowing for simple discrete and continuous dynamics in this model the worksheet simulates the time paths of national income and interest rate.

The IS-Curve

The aggregate demand for goods equals production:

$$y = X + c \cdot y - b \cdot i$$

with
- y : national income, production
- X : autonomous spending
- i : interest rate
- c : marginal propensity to consume
- b : marginal propensity to invest

Solving this equation for $i = IS(y)$ yields:

$$IS(y) := \frac{X - y + c \cdot y}{b}$$

The LM-Curve

The money demand is a function of income and interest rate. The money market reaches its equilibrium if the money supply M equals money demand:

$$M = v \cdot y - w \cdot i \qquad \text{with } v, w > 0$$

Solving this equation for $i = LM(y)$ yields:

$$LM(y) := \frac{v \cdot y - M}{w}$$

The Static Equilibrium

The simultaneous equilibrium of the goods and money markets is determined by the intersection point of the IS- with the LM-curve. Therefore, the solution of

$$\frac{v \cdot y - M}{w} - \frac{X - y + c \cdot y}{b} = 0$$

yields the equilibrium value y_E of income:

$$y_E := \frac{(M \cdot b + X \cdot w)}{(v \cdot b + w - c \cdot w)}$$

To get the equilibrium interest rate i_E, insert y_E into the LM- (or IS-)function:

$$i_E := LM(y_E)$$

Discrete Dynamics

Assumption: Starting with initial values $y_0 := y_{init}$ and $i_0 := i_{init}$ the change of national income and interest rate from period "time" to "time+1" depends on the deviation between planned and actual values in period "time":

$$y_{time+1} - y_{time} = \delta_1 \cdot \left(X + c \cdot y_{time} - b \cdot i_{time} - y_{time}\right)$$

$$i_{time+1} - i_{time} = \delta_2 \cdot \left(v \cdot y_{time} - w \cdot i_{time} - M\right)$$

where δ_1 and δ_2 are speed of adjustment parameters.

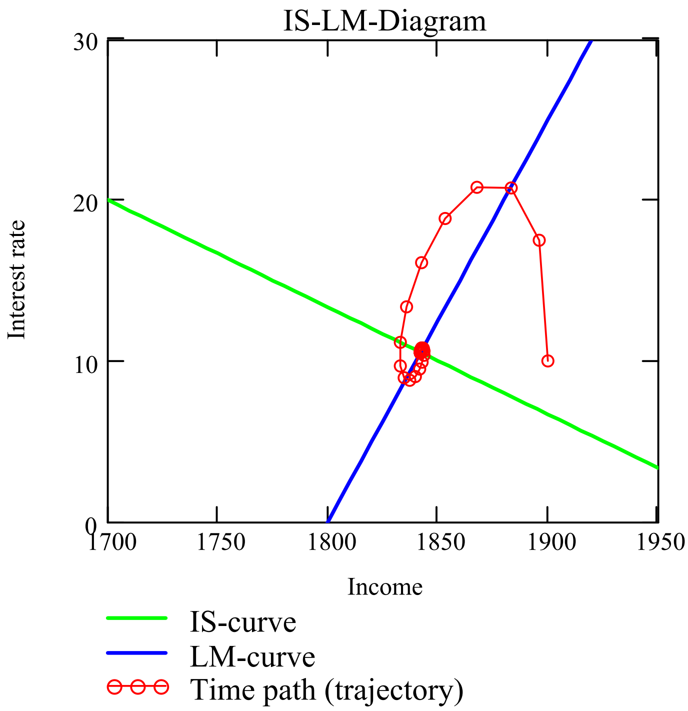
This is a system of linear difference equations in two variables. For $time := 0 .. T_{max}$ we solve this system to generate the time path of the first T_{max} periods, writing:

$$\begin{bmatrix} y_{time+1} \\ i_{time+1} \end{bmatrix} := \begin{bmatrix} \delta_1 \cdot \left(X + c \cdot y_{time} - b \cdot i_{time} - y_{time}\right) + y_{time} \\ \delta_2 \cdot \left(v \cdot y_{time} - w \cdot i_{time} - M\right) + i_{time} \end{bmatrix}$$

Numerical Example

At first we use the parameters of Eckalbar (1993), who wrote his example in the programming language *MATHEMATICA*:

Parameters of the IS-curve:	$X \equiv 400$	$c \equiv 0.8$	$w \equiv 1$
Parameters of the LM-curve:	$M \equiv 450$	$b \equiv 3$	$v \equiv .25$
Adjustment parameters:	$\delta_1 \equiv .4$	$\delta_2 \equiv .5$	
Initial values:	$y_{init} \equiv 1900$	$i_{init} \equiv 10$	
Number of periods:	$T_{max} \equiv 50$		



Plot range:

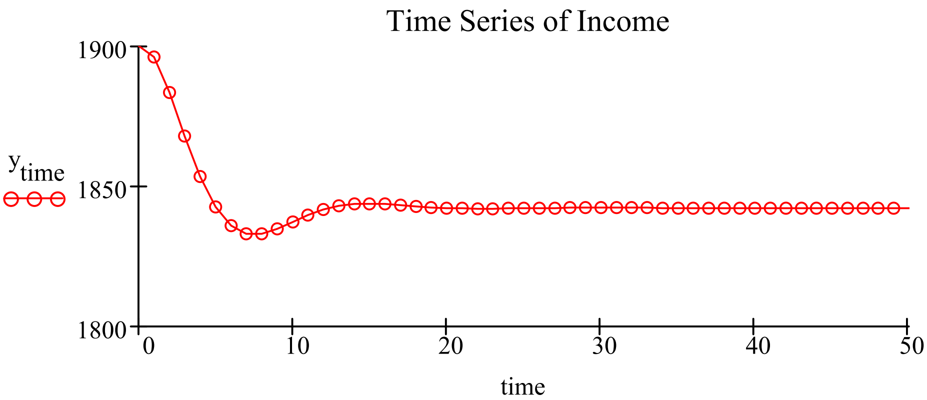
ymin≡ 1700 ymax≡ 1950

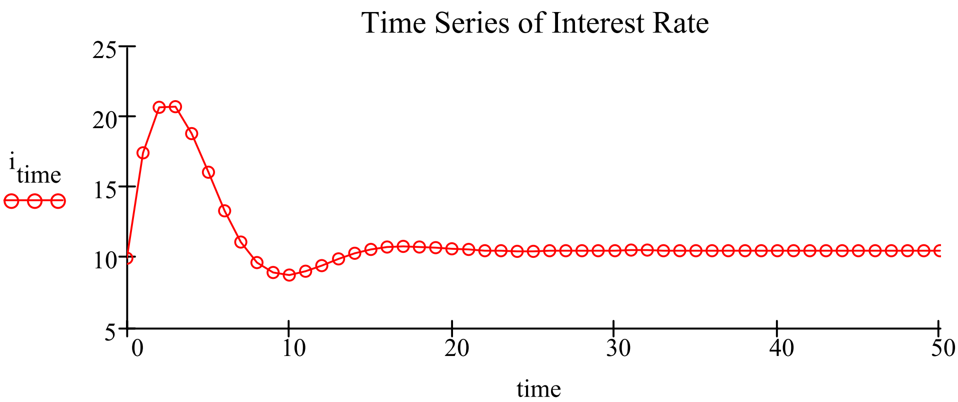
imin≡ 0 imax≡ 30

Equilibrium:

$y_E = 1842.105$

$i_E = 10.526$





Stability condition: It can be shown that the equilibrium is asymptotically stable if and only if the absolute values of the eigenvalues of the matrix of parameters below are less than one:

$$\overrightarrow{\left|\text{eigenwerte}\left[\begin{bmatrix}\delta_1\cdot(c-1)+1 & -\delta_1\cdot b \\ \delta_2\cdot v & 1-\delta_2\cdot w\end{bmatrix}\right]\right|} = \begin{bmatrix}0.781 \\ 0.781\end{bmatrix}$$

Continuous Dynamics

Consider now the continuous time linear differential equation system:

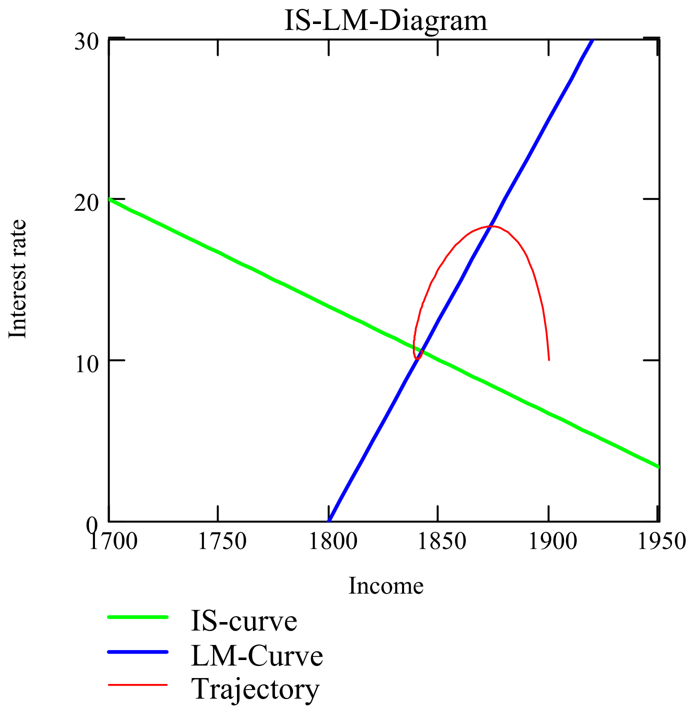
$$\frac{d}{d\text{time}}y = \delta_1\cdot(X + c\cdot y - b\cdot i - y)$$

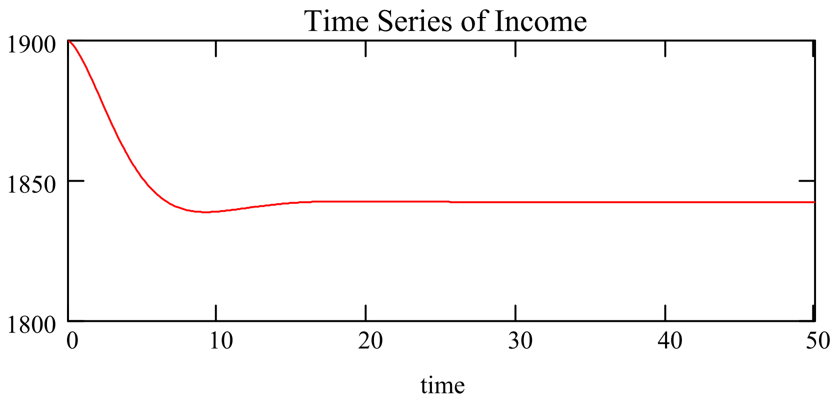
$$\frac{d}{d\text{time}}i = \delta_2\cdot(v\cdot y - w\cdot i - M)$$

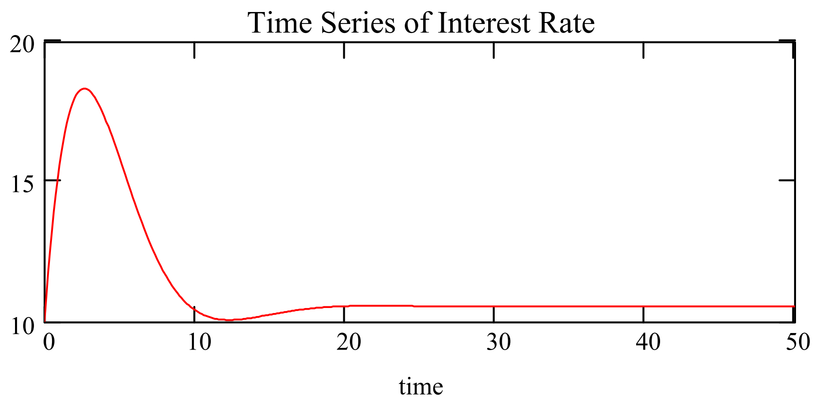
$$x_0 := y_{init} \qquad x_1 := i_{init}$$

Because we use the identical parameters as before, the IS-curve, the LM-curve and the simultaneous equilibrium will be the same as in the discrete case. But the trajectories become continuous and may differ from discrete dynamics. Now the equilibrium is asymptotically stable if the real parts of the eigenvalues of the parameter matrix below are negative:

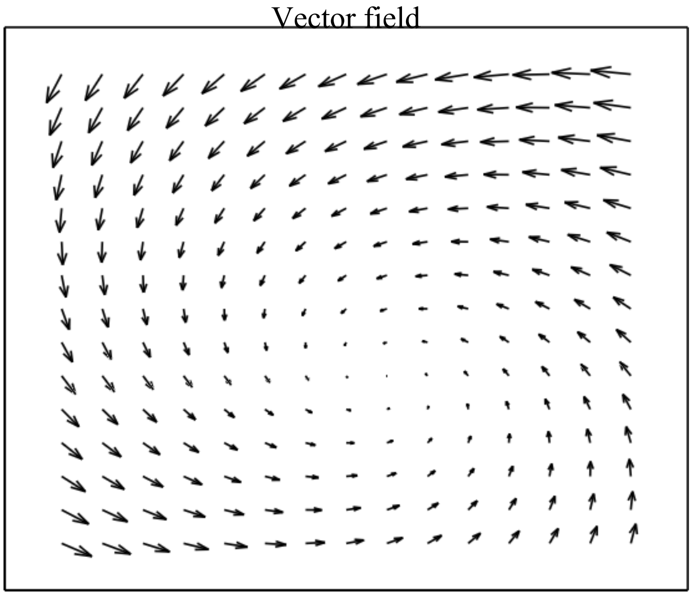
$$\text{eigenwerte}\left[\begin{bmatrix}\delta_1\cdot(c-1) & -\delta_1\cdot b \\ \delta_2\cdot v & -\delta_2\cdot w\end{bmatrix}\right] = \begin{bmatrix}-0.29+0.325i \\ -0.29-0.325i\end{bmatrix}$$


IS-LM-Diagram
Interest rate
Income
0
10
20
30
1700
1750
1800
1850
1900
1950
IS-curve
LM-Curve
Trajectory


Time Series of Income
1800
1850
1900
0
10
20
30
40
50
time


Time Series of Interest Rate
10
15
20
0
10
20
30
40
50
time

The vector field for this system gives a fairly complete picture of the dynamics around the equilibrium.



$imax = 30$

$imin = 0$

$(income, interest\_rate)$

$ymin = 1700$ $ymax = 1950$

It's Your Turn!

1. Try a rapid adjustment of the money market but a rather slow adjustment of the aggregate demand of goods (for example: $\delta_1 = 0.05$ and $\delta_2 = 1$).
2. Now let $\delta_1 = 0.9$ and $\delta_2 = 0.9$ and decrease w (for example let w = 1, 0.8, 0.7, 0.6, 0.579. and 0.5.). Compare stability and trajectories between the discrete and the continuous case.

Literature:

Eckalbar, J.C.: Economic Dynamics. In: Varian, H.R. (ed.): Economics and Financial Modeling with Mathematica. Santa Clara, CA., 1993, p. 63 - 68.