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A Nelson/Winter-Model of Schumpeterian Competition

Summary:

Nelson and Winter were among the first to translate Schumpeter's verbal account of economic evolution into the formal description of a Markov process allowing for numerical simulations. An important role plays the distinction between enterpreneurs responsible for innovation, and the imitators responsible for diffusion. At the beginning of each market simulation the firms have an identical productivity, the same level of unit cost and an equal share of the market. One half of the firms has a potential capacity for innovation and imitation. The other firms are exclusively imitators. The human knowledge follows an exogenous trend. This trend may be a constant growth rate of latent productivity or exhibits a logistic form. The logistic form of the productivity evolution was introduced by Caccomo (1996). In this case the market dynamics obey a life cycle in which three traditional phases can be distinguished: the exploration phase, the takeoff phase and the saturation phase.

Market behaviour in the short run

The production of each firm is characterised by constant returns to scale. The output is decided by a full-capacity utilisation rule (⇔ supply = maximum output).

Individual supply of firm $i=1,2..n$ at date "time" is: $q_{i,time}=a_{i,time}\cdot K_{i,time}$

where

$a_{i,time}$: productivity of capital of firm i at date "time"

$K_{i,time}$: capital stock of firm i at date "time"

Total supply: $$Q_{time}=\sum_{i=1}^{n} q_{i,time}$$

Inverse market demand function (short term equilibrium price):

$$p_{time}=\frac{D}{Q_{time}} \qquad \text{with } D>0$$

Profit per unit of capital for firm i at date "time":

$$\pi_{i,time}=p_{time}\cdot a_{i,time}-c-r_{im}-r_{in}$$

where

c : acquisition cost per unit of capital

r_{im}: cost of imitative R&D per unit of capital

r_{in}: cost of innovative R&D per unit of capital

Technological progress

The mean productivity of process innovations $amean_{time}$ in a given period is determined by an exogenously given state of public knowledge.

Case 1: The knowledge grows through time with the constant rate ϕ:

$$amean_{time} = amean_{time-1} \cdot (1 - \phi)$$

Case 2: The growth of knowledge evolves as a logistic time function:

$$amean_{time} = \frac{\left(B - a_{init}\right)}{1 + \exp(\beta - \phi \cdot time)} + a_{init}$$

where

a_{init} :	initial value of productivity
B :	the maximum level of latent productivity
β :	parameter that conditions the takeoff period
ϕ :	parameter that affects the growth rate of the latent productivity after the takeoff

R&D generates random results. Innovation is a two-stage stochastic phenomenon. A first random draw determines if the R&D investement of the firm has been successful. This draw follows a Poisson-distribution, where the success probability depends on the R&D expenditure $r_{in} \cdot K_{i,time}$ and the internal sources d_{in} of innovation:

$$draw_{i,time} \leftarrow rpois\left(1, d_{in} \cdot r_{in} \cdot K_{i,time}\right)$$

A second draw gives the effective result of the innovation (ain). This draw follows a Lognormal-distribution with standard deviation σ:

$$ain_{i,time} \leftarrow e^{rnorm\left(1, \ln\left(amean_{i,time}\right), \sigma\right)}$$

For the imitation, we have only one random draw which determines if the firm's R&D investment has been successful. This draw follows a Poisson-distribution, where the success probability depends on the imitation R&D expenditure $r_{im} \cdot K_{i,time}$ and the external sources d_{im} of imitation:

$$draw_{i,time} \leftarrow rpois\left(1, d_{im} \cdot r_{im} \cdot K_{i,time}\right)$$

If the draw is successful the imitator obtains the best practice in the industry:

$$amax = max\left(a_{time}\right)$$

We assume, that a firm $i > \frac{n}{2}$ is only a pure imitator. Finally the new productivity of a firm for the next period is given by the best of these three outcomes:

$$a_{i,time+1} = max\left(amax, a_{i,time}, ain_{i,time}\right)$$

Investment behaviour

The maximum investment $I_{max_{i,time}}$ (per capital unit) of a firm is determined by the rate of depreciation δ, the profits of the present period plus the loans from the bank in proportion to this profits:

$$I_{max_{i,time}} = \delta + \pi_{i,time} + max\left(\begin{bmatrix} 0 & b \cdot \pi_{i,time} \end{bmatrix}\right) \qquad \text{with} \qquad b \geq 0$$

Firms adjust their capital stock in accordance with their "Cournot conjectures". Desired investment results from the comparison of the actual margin of the firm with its target margin influenced by the market share of the firm. If the actual mark-up is higher than the desired mark-up the firm considers to increase the output. Consequently its capital stock must increase and the investment will over-compensate the depreciation of capital:

desired mark-up:

$$mu_{des_{i,time}} = \frac{2 \cdot \eta - \dfrac{q_{i,time}}{Q_{time}}}{2 \cdot \eta - 2 \cdot \dfrac{q_{i,time}}{Q_{time}}}$$

where $\eta \geq 0$ is the lack of agressiveness in investment strategy.

actual mark-up: $$\mathrm{mu\ act}_{i,time} = \frac{p_{time} \cdot a_{i,time+1}}{c}$$

desired investment: $$\mathrm{I\ des}_{i,time} = \delta + 1 - \frac{\mathrm{mu\ des}_{i,time}}{\mathrm{mu\ act}_{i,time}}$$

Now we sum up the investment decision

$$\mathrm{Inv}_{i,time} = \max\left(0, \min\left(\mathrm{I\ max}_{i,time}, \mathrm{I\ des}_{i,time}\right)\right)$$

and calculate the capital available at the beginning of the next period:

$$K_{i,time+1} = K_{i,time} \cdot \left(\mathrm{Inv}_{i,time} + 1 - \delta\right)$$

Given the transition law of the system (i.e. the yellow marked equations from above) we have a non-stationary Markov system which entails simulation studies.

Parameters

Initial value of productivity: $a_{init} := 0.16$

Ratio of external financing to economic profit: $b := 1$

Production cost per unit of capital: $c := 0.16$

Depreciation rate per period: $\delta := 0.03$

Total revenue of industry: $D := 67$

Lack of aggressiveness in investment strategy: $\eta := 1$

Number of firms ($n \le 10$): $n := 4$

Number of time periods: $T_{max} := 100$

Cost of imitative R&D: $TR_{im} := .4$

Cost of innovative R&D: $TR_{in} := 4.0$

A firm's "internal sources" to get an innovative draw: $d_{in} := .125$

A firm's "external sources" to get an imitative draw: $d_{im} := 1.125$

Standard deviation : $\sigma := 0.01$

Case number (= 1 or 2) of science based innovation trend: $trend := 1$

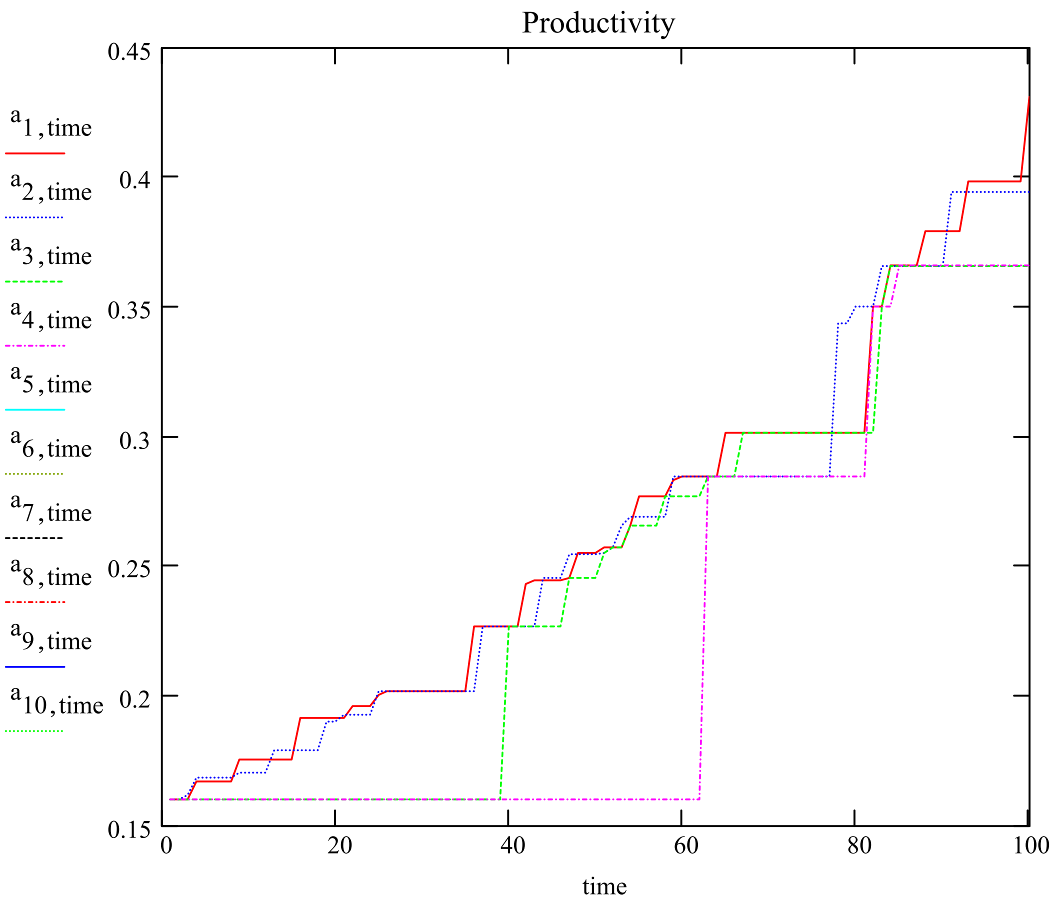
Rate of latent productivity increase: $\phi := 0.01$

Maximum level of latent productivity: $B := .5$

Parameter that conditions the takeoff period: $\beta := 7$

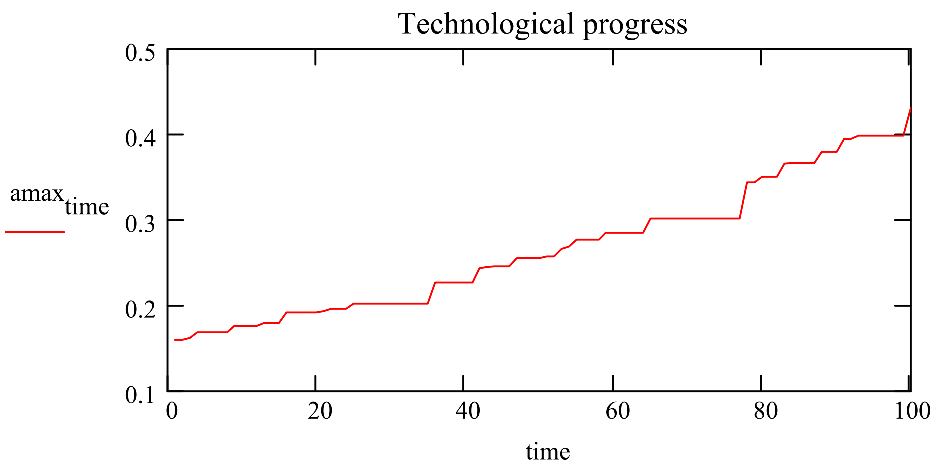
Simulation results

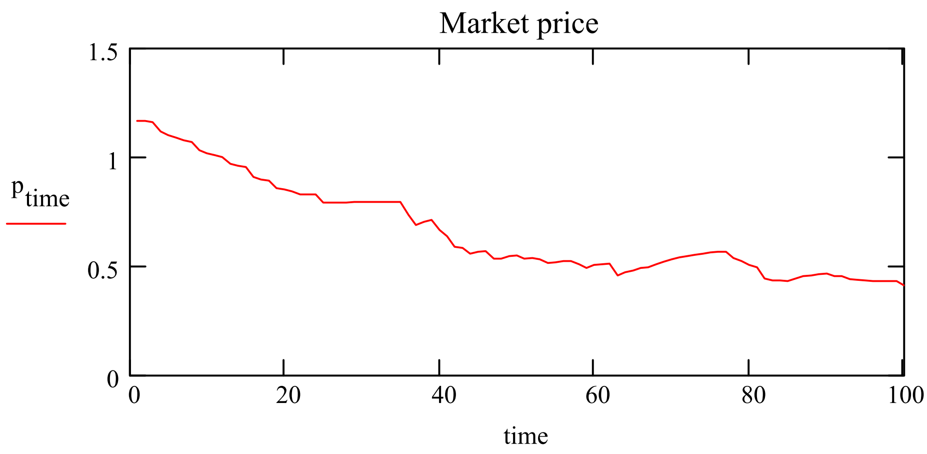
To run a new simulation choose "Compute Worksheet" from the *Mathcad* menue above. Every time you do this, another random selection of R&D will be generated.



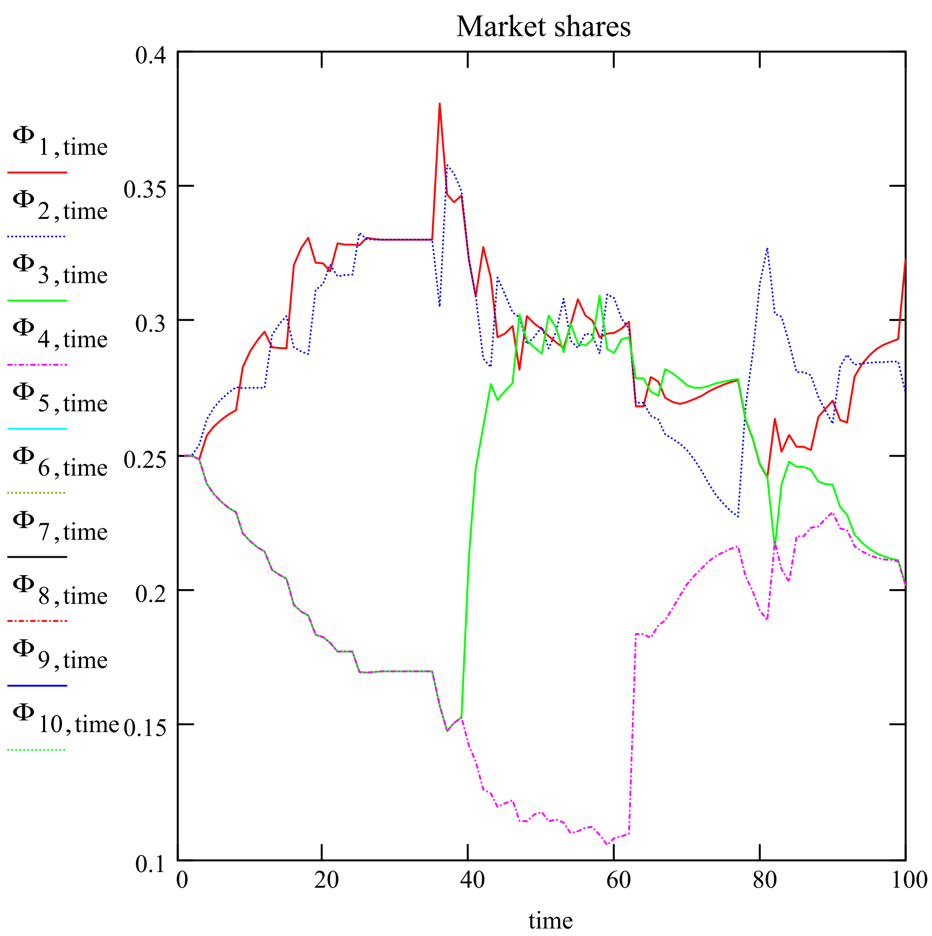
The realized technological progress corresponds to the best practice:

$$amax_{time} := \max\left(a^{\langle time \rangle}\right)$$



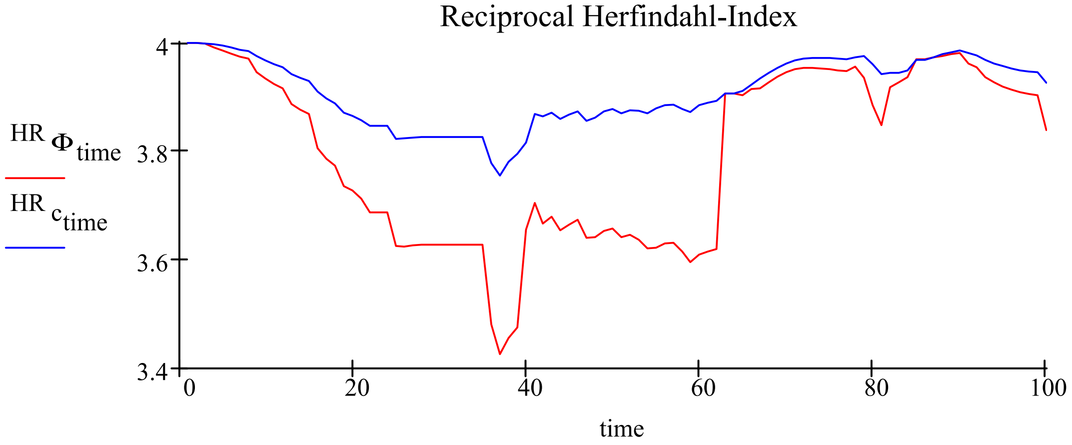


Market share of firm i at period "time": $\Phi_{i,time} := \frac{q_{i,time}}{Q_{time}}$



As a measure of concentration of turnover and capital on the market we use the **reciprocal Herfindahl-Index (HR)**

$$HR\Phi_{time} := \frac{1}{\sum_{j=1}^{n} \left(\Phi_{j,time}\right)^2} \qquad HRc_{time} := \frac{1}{\sum_{j=1}^{n} \left[\frac{c_{j,time}}{\sum_{j=1}^{n} c_{j,time}}\right]^2}$$



It' s Your Turn!!

1. Let ceteris paribus n = 8, β = 1, σ = 0.001. Thus the exploration phase is non existent and the takeoff is immediate. The selection process intensifies.
2. Select ceteris paribus "trend = 2" and T_{max} = 2000. Vary the ability of imitation.

(Note: Although the number of firms may be greater than 10, the graph presentation has strict limitations.)

Literature :

Anderson, E.S. et al.: The Nelson and Winter Models Revisited: Prototypes for Computer-Based Reconstruction of Schumpeterian Competition. DRUID Working Paper No. 96-2. Danish Resarch Unit for Industrial Dynamics. Aalborg, 1996.

Caccomo, J.-L.: Technological evolution and economic instability: theoretical simulations. Evolutionary Economics, Vol. 6 (1996), 141 - 155.

Nelson, R./Winter, S.: An evolutionary theory of economic change. Cambridge, 1982.