FH-Kiel
University of Applied Sciences

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A Ricardian Model with Very Many Goods

Summary:

In the Ricardian model the trade pattern between two countries is determined by comparative advantage. The simple text book version of this model is restricted to the case of two commodities. The well known article by Dornbusch/Fischer/Samuelson (1977) develops a Ricardian model in which the number of goods becomes very large, so that one can think of them as a continuum of commodities. Using their model, the effects of supply shocks and trade policy to the pattern of specialization are simulated.

1. Important Definitions

A(z):	comparative costs for good z
$a_i(z)$:	unit labour requirement of good z in country i = 1,2
$Cons_i(z)$	consumption of good z in country i
δ_i:	import tariff rate
L_i:	available labour units in country i
$p_i(z)$:	(money) price of good z in country i
w_i:	(money) wage rate in country i
ω:	relative wage rate
Y_i:	total income in country i
z:	commodity index, where $0 \le z \le 1$

2. The supply side of the model

Suppose that two countries i = 1,2 have only one factor of production: labour (L_i). Each country consumes and is able to produce an extremly large number of goods. In the case of a continuum of goods, we can index commodities with an index number $z \in [0,1]$. The labour requirement of a commodity z in country i is $a_i(z)$. The relative labour costs (**comparative costs**) are then defined for each good as:

$$A(z) = \frac{a_1(z)}{a_2(z)}$$

It is assumed, that commodities are ranked in monotonically decreasing order of country 2's comparative advantage; thus:

$$A(0) \ge A(z) \ge A(1) \quad \text{and} \quad \frac{d}{dz}A(z) < 0$$

For a numerical example let:

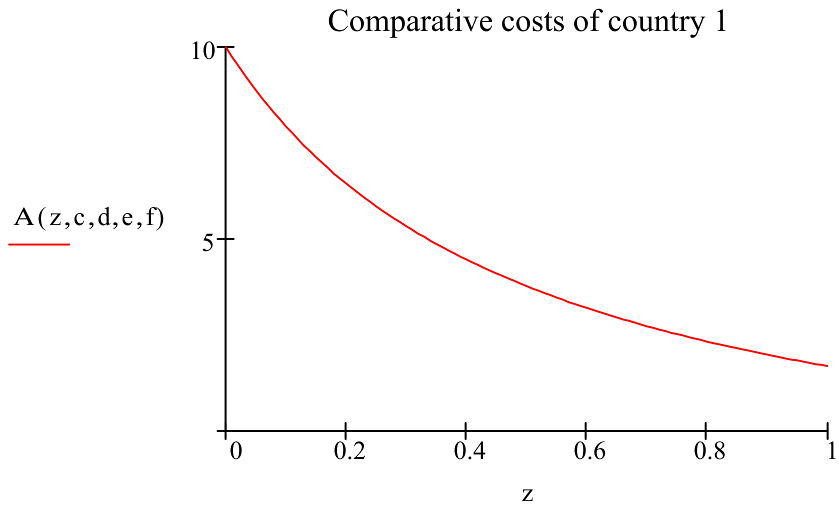
$a_1(z,c,d) := c - d \cdot z$ with $c, d > 0$ $c := 10$ $d := 5$

$a_2(z,e,f) := e + f \cdot z$ with $e, g > 0$ $e := 1$ $f := 2$

Then:

$$A(z,c,d,e,f) := \frac{a_1(z,c,d)}{a_2(z,e,f)} \rightarrow \frac{(c - d \cdot z)}{(e + f \cdot z)}$$

$z := 0, .01 .. 1$



Any commodity z will be produced in country 2 if:

$a_2(z) \cdot w_2 \le a_1(z) \cdot w_1$

Hence, if

$\omega = \frac{w_2}{w_1} < \frac{a_1(z)}{a_2(z)} = A(z)$ $\Rightarrow$ Commodity z is produced in country 2.

$\omega = \frac{w_2}{w_1} > \frac{a_1(z)}{a_2(z)} = A(z)$ $\Rightarrow$ Commodity z is produced in country 1.

For international trade to be possible it is obviously required that

$A(1) < \omega < A(0)$

In our numerical example these limits are:

$A(1, c, d, e, f) = 1.667$

$A(0, c, d, e, f) = 10$

Because of the continuity assumption there exists a **marginal good** Z for which

$\omega = A(Z)$

such that country 2 will produce all commodities in the interval

$0 \le z < Z$

more efficiently than country 1, while all commodities in the interval

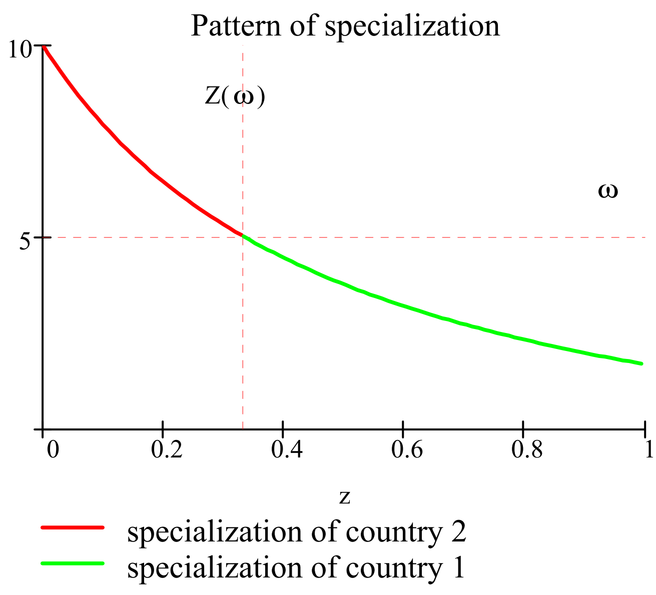
$Z < z \le 1$

will be produced more efficiently by country 1. We can determine the marginal good from the inverse function of $A(z)$. For our numerical example this is:

$$Z(\omega) := \omega = A(z,c,d,e,f) \text{ auflösen}, z \rightarrow \frac{-(\omega - 10)}{(2\cdot\omega + 5)}$$

Now choose a relative wage rate, for example $\omega := 5$

The next figure characterizes the efficient geographic specialization. Change ω to see, how this mechanism of specialization works.



3. The demand side of the model

Assuming identical tastes with homothetic utility functions, the structure of demand is identical in both countries and independent of the level of income. If $b_i(z)$ denotes the **budget share** of commodity z in country i, then

$$b_1(z)=b_2(z)=b(z)$$

where

$$\int_0^1 b(z)\,dz=1$$

For further numerical computations we assume the following distribution of budget shares:

$$b(z) := 2 - 2\cdot z$$

Check:

$$\int_0^1 b(z)\,dz = 1$$

We denote the fraction of income spent in each country (and hence in the "world") on those goods for which country 2 has a comparative advantage as:

$$\nu(Z) := \int_0^Z b(z)\,dz \rightarrow 2\cdot Z - Z^2$$

Hence $1-\nu(Z)$ is the fraction of world income spent on the commodities for which country 1 has a comparative advantage.

In a perfectly competitive setting, labour is fully employed, and the value of output coincides with total income. Thus, total income of country i results from:

$$Y_i=w_i\cdot L_i$$

Now we obtain the condition of the **balance of trade equilibrium**:

$$(1-\nu(Z))\cdot Y_2=\nu(Z)\cdot Y_1 \qquad \Leftrightarrow \qquad (1-\nu(Z))\cdot w_2\cdot L_2=\nu(Z)\cdot w_1\cdot L_1$$

$$\Rightarrow \qquad \omega=\frac{w_2}{w_1}=\frac{L_1}{L_2}\cdot\frac{\nu(Z)}{1-\nu(Z)}$$

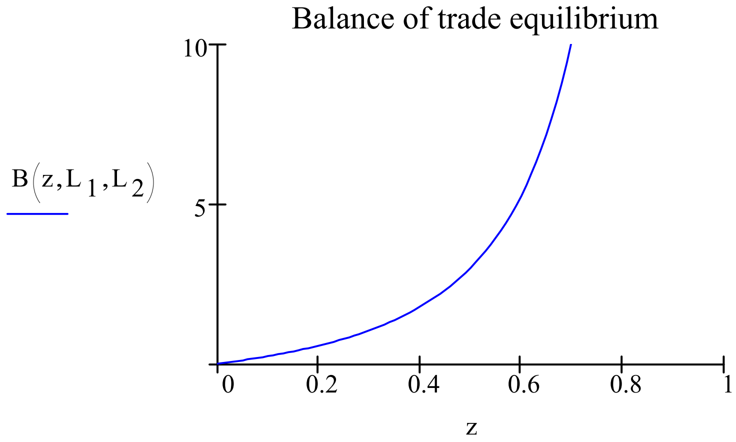
This condition enables us to associate with each Z a value of ω such that the world market clears. Let for example

$L_1 := 10$ $\qquad$ $L_2 := L_1$

then the monotonically increasing schedule

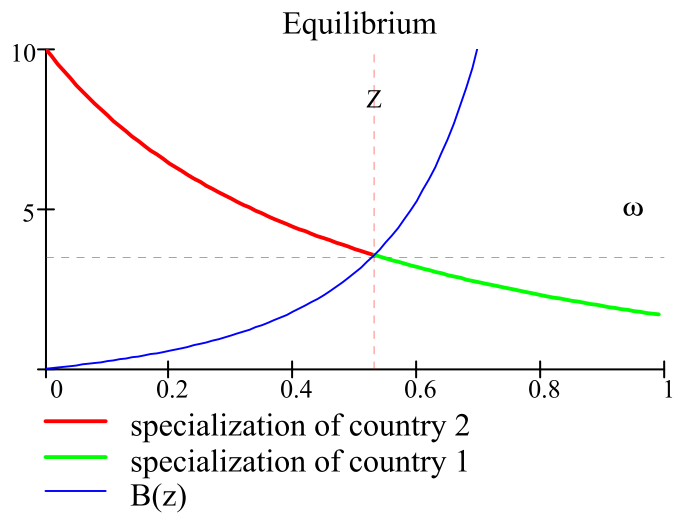
$$B(Z, L_1, L_2) := \frac{L_1}{L_2} \cdot \frac{\nu(Z)}{1 - \nu(Z)}$$

figures this interrelation:



4. Determination of the relative wage rate and the pattern of specialization

To determine the relative wage ω in equilibrium and the marginal good Z simultaneously, we have to compute the intersection point of the A(z)- with the B(z)-curves.



$Z = 0.532$

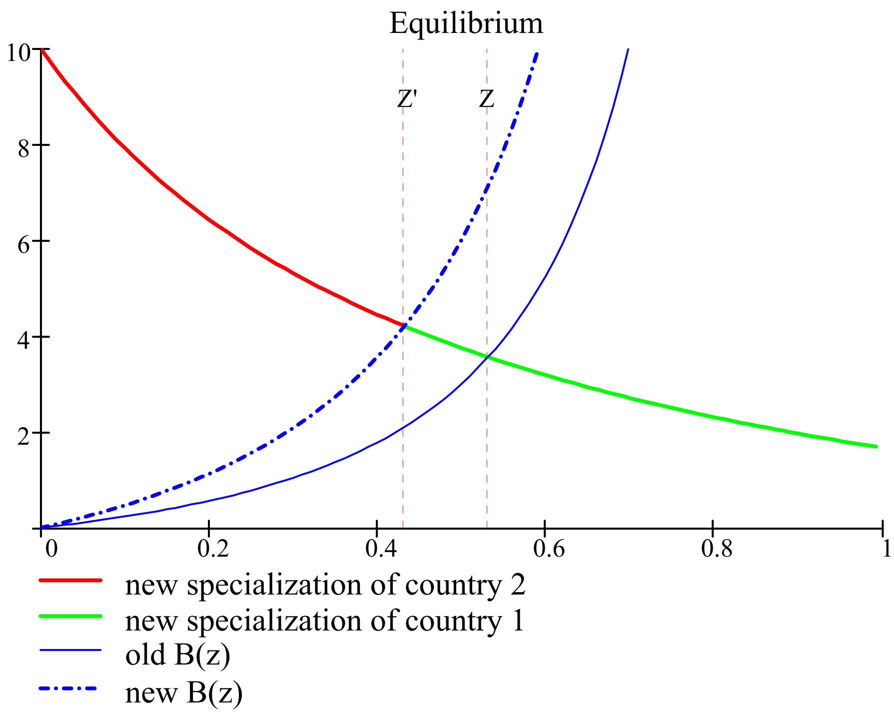
$\omega = 3.558$

5. Simulation Runs

Case I: Effects of a variation of the labour force in country 2

Enter a new value of the labour force in country 2: $L_{2new} := L_1 \cdot 0.5$

Now a new equilibrium (Z', ω') is computed. Notice the shift of the B(z)-curve.



Old equilibrium:

$Z = 0.532$

$\omega = 3.558$

New equilibrium:

$Z' = 0.432$

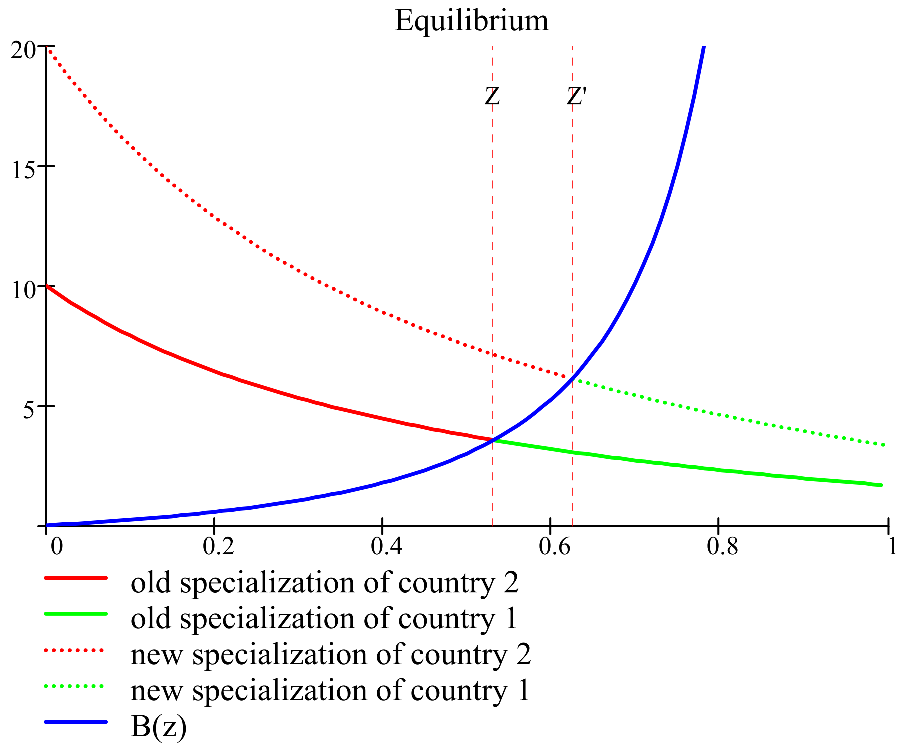
$\omega' = 4.204$

Case II: Productivity shock in country 2

To rise productivity in country 3 decrease the parameters e and/or f.

$e_{new} := .5$ $f_{new} := 1$

Notice the shift of the A(z)-curve.



Old equilibrium:

$Z = 0.532$

$\omega = 3.558$

New equilibrium:

$Z' = 0.625$

$\omega' = 6.111$

Case 3: Trade policy [borrowed from Gandolfo (1998, Appendix A.3.4)]

Assumption: Each country i levies a uniform tariff rate δ_i on all imports. The proceeds are rebated in lump sum form.

Then the price of good z from country 2 in country 1 is

$$\left(1+\delta_1\right)\cdot p_2(z) = \left(1+\delta_1\right)\cdot a_2(z)\cdot w_2$$

and the price of good z from country 1 in country 2 is

$$\left(1+\delta_2\right)\cdot p_1(z) = \left(1+\delta_2\right)\cdot a_1(z)\cdot w_1$$

Hence any commodity z will be produced in country 1 if

$$a_1(z)\cdot w_1 \le \left(1+\delta_1\right)\cdot a_2(z)\cdot w_2$$

namely

$$A(z) \le \omega\cdot\left(1+\delta_1\right)$$

and similarly, for those goods produced by country 2:

$$A(z) \ge \frac{\omega}{1+\delta_2}$$

It follows that there are **two marginal goods**. Denote Z_i the marginal commodity from the point of view of country i. Define λ_i as the fraction of country i's income spent on goods produced on the same country:

$$\lambda_1\left(Z_1\right) := \int_{Z_1}^{1} b(z)\,dz$$

$$\lambda_2\left(Z_2\right) := \int_{0}^{Z_2} b(z)\,dz$$

Now the expenditure Y_i in every country i includes the lump-sum tariff rebates:

$$Y_i = w_i\cdot L_i + \delta_i\cdot\left[\left(1-\lambda_i\left(Z_i\right)\right)\cdot\frac{Y_i}{1+\delta_i}\right] = \frac{w_i\cdot L_i\cdot\left(1+\delta_i\right)}{1+\lambda_i\left(Z_i\right)\cdot\delta_i} \qquad \text{with} \qquad i = 1,2$$

Therefore, the **trade balance equilibrium** is

$$\frac{\left(1-\lambda_1\left(Z_1\right)\right)\cdot Y_1}{1+\delta_1}=\frac{\left(1-\lambda_2\left(Z_2\right)\right)\cdot Y_2}{1+\delta_2}$$

Now we set up an equation system to let *Mathcad* search for the new equilibrium:

Guess values to start the numerical solution are: $Z_1 := Z \quad Z_2 := Z \quad \omega' := \omega$

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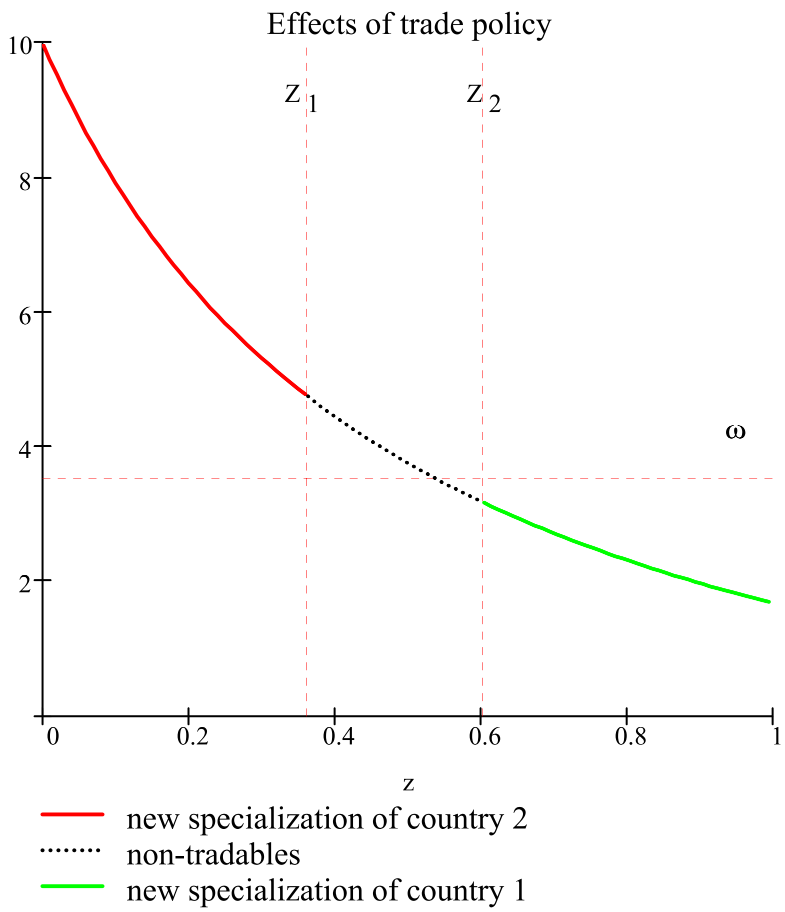
$$\frac{\left(1-\lambda_1\left(Z_1\right)\right)\cdot\frac{L_1\cdot\left(1+\delta_1\right)}{1+\lambda_1\left(Z_1\right)\cdot\delta_1}}{1+\delta_1}=\frac{\left(1-\lambda_2\left(Z_2\right)\right)\cdot\frac{\omega'\cdot L_2\cdot\left(1+\delta_2\right)}{1+\lambda_2\left(Z_2\right)\cdot\delta_2}}{1+\delta_2}$$

$$A\left(Z_1,c,d,e,f\right)=\omega'\cdot\left(1+\delta_1\right)$$

$$A\left(Z_2,c,d,e,f\right)=\frac{\omega'}{1+\delta_2}$$

$$\begin{bmatrix}\omega'\\ Z_1\\ Z_2\end{bmatrix} := \text{Suchen}\left(\omega',Z_1,Z_2\right)$$

Enter tariff rates: $\delta_1 \equiv 0.5$ $\delta_2 \equiv 0$



relative wage rate without tariffs:

$\omega = 3.558$

relative wage rate with tariffs:

$\omega' = 3.159$

marginal goods:

$Z_1 = 0.363$

$Z_2 = 0.604$

Notice: The presence of a tariff gives rise to a range $Z_1 \le z \le Z_2$ of non-traded commodities!

Try to simulate the effects of a "tariff war" between both countries.

Literature:

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