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Sohmen's Trade Balance Equation and Complex Dynamics of the Real Exchange Rate

Summary:

Fluctuations in real exchange rates are mostly ascribed to fluctuations in underlying economic "fundamentals". But Chen (1999) shows that a modification of a simple deterministic classical trade model with an S-shape trade balance curve introduced by Sohmen (1957) can generate complex endogenous dynamics of the real exchange rate. The precondition for irregular cycles to emerge is that the Marshall-Lerner condition is not satisfied and that capital mobility is imperfect. The conclusion therefore is that the short run volatility of the real exchange rate can be eliminated by raising the capital mobility.

Assumptions

- The **price of foreign ouput** is given: $p_F = 1$

- Domestically produced goods differ from foreign goods. The **supply** of domestic goods is fixed.
- The **price p of domestic output** is perfectly flexible.

- The **real exchange rate** (terms of trade) is defined as: $q = \frac{e \cdot p_F}{p} = \frac{e}{p}$ where e is the nominal exchange rate (domestic currency price of foreign currency).

- The **domestic bond rate** of return in terms of domestic goods r and the foreign bond rate of return in terms of foreign goods i_F are assumed to be constant and equal: $r = i_F$

- Under the condition of perfect foresight, the expected gross yield on the foreign bond expressed in domestic goods at period "time" is

$$1 + r_{F_{time}} = \left(1 + i_F\right) \cdot \frac{q_{time+1}}{q_{time}}$$

where r_F denotes the rate of return in terms of domestic goods on foreign bonds.

- The **capital flows** (in terms of domestic goods) are proportional to ($r_{F_{time}} - r_{time}$):

$$q_{time} \cdot K_{time} = \phi \cdot \left(r_{F_{time}} - r_{time}\right)$$

where K_{time} denotes net capital outflow in terms of foreign goods and where ϕ represents the **speed of adjustment** in asset markets.

- Domestic demand for **imports**: $IM\left(q_{time}\right) = a - b \cdot q_{time}$ with $a, b > 0$

- Foreign demand for home **exports**: $EX\left(q_{time}\right) = c - \frac{d}{q_{time}}$ with $c, d > 0$

- Let TB denote the **trade balance** in terms of foreign goods. Thus, we obtain the S-shape trade balance function from Sohmen (1957):

$$TB\left(q_{time}\right) = \frac{EX\left(q_{time}\right)}{q_{time}} - IM\left(q_{time}\right) = \frac{c}{q_{time}} - \frac{d}{\left(q_{time}\right)^2} - a + b \cdot q_{time}$$

- Under a flexible exchange rate regime, the **balance of payments** sum to zero. Thus, the equilibrium condition for the balance of payments is

$$TB\left(q_{time}\right) - K_{time} = 0$$

Short term dynamics of the real exchange rate

After some substituting we obtain:

$$q_{time} \cdot TB\left(q_{time}\right) - \phi \cdot \left(1 + i_F\right) \cdot \left(\frac{q_{time+1}}{q_{time}} - 1\right) = 0$$

Rearrangements yield a **first-order nonlinear difference equation** in real exchange rates:

$$q_{time+1} = q_{time} + \frac{\left(q_{time}\right)^2 \cdot TB\left(q_{time}\right)}{\phi \cdot \left(1 + i_F\right)} \qquad \Leftrightarrow$$

$$q_{time+1} = q_{time} + \frac{1}{\phi \cdot \left(1 + i_F\right)} \cdot \left[b \cdot \left(q_{time}\right)^3 - a \cdot \left(q_{time}\right)^2 + c \cdot q_{time} - d\right]$$

Equilibria and stability

In the stationary state, $q_{time+1} = q_{time} = q_E$ the difference equation from above implies the cubic equation:

$$b \cdot \left(q_E\right)^3 - a \cdot \left(q_E\right)^2 + c \cdot q_E - d = 0$$

Therefore, the equilibrium values of the real exchange rate are independent of ϕ and i_F.

To compute a numerical example let

$\mu = \phi \cdot \left(1 + i_F\right)$ with $\mu := 0.36$

$a := 7$ $\quad b := 2$ $\quad d := 2$

$c := a - b + d$ $\quad \Leftarrow$ (Without loss of generality under this condition one root of the cubic equation is normalized to 1.)

There exist three distinct positive roots, as long as:

$(a>2\cdot b + d) = 1$ $\Leftarrow$ (1 = condition is satisfied)

We obtain the **solution**:

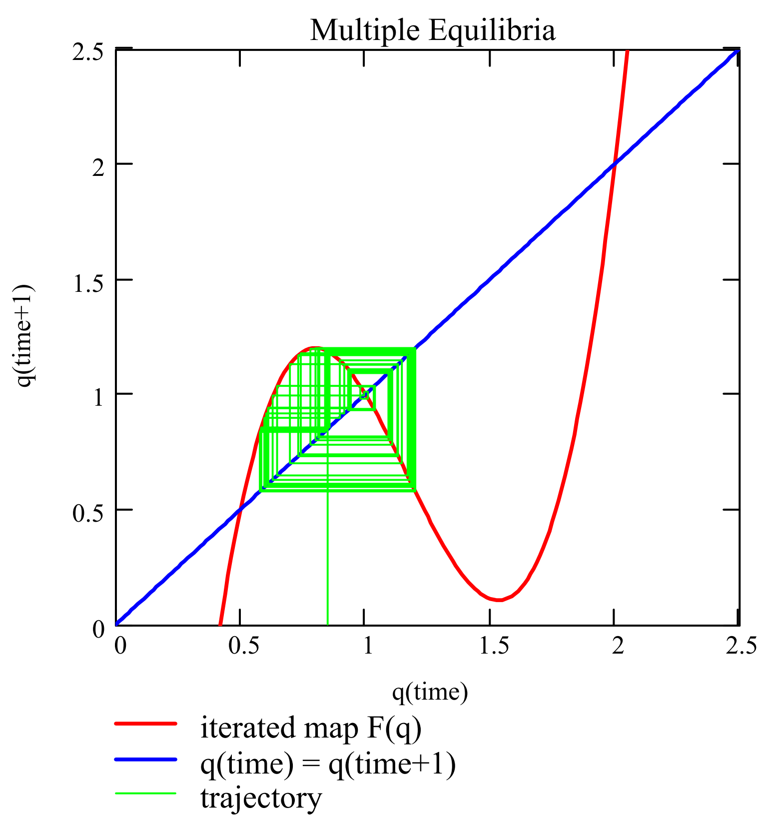
$$q_E := \left[b\cdot\left(q_E\right)^3 - a\cdot\left(q_E\right)^2 + c\cdot q_E - d=0\right] \text{ auflösen}, q_E \rightarrow \begin{bmatrix} 1 \\ 2 \\ \frac{1}{2} \end{bmatrix}$$

The difference equation of the real exchange rate is written as an **iterated map**:

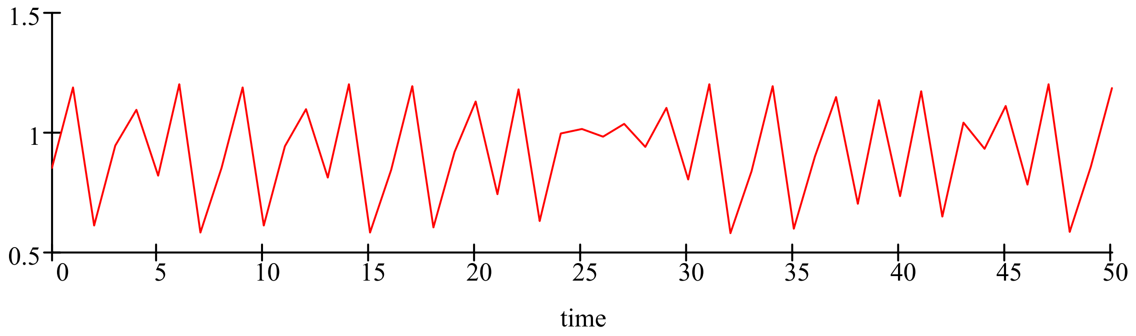
$$F(q,\mu) := q + \frac{1}{\mu}\cdot\left(b\cdot q^3 - a\cdot q^2 + c\cdot q - d\right)$$

The figure below shows multiple equilibria as intersection points of this iterated map with the 45° line. Given a maximum number of periods $T_{max} := 50$ and an initial value of the real exchange rate the trajectory of the time path of the exchange rate is presented as a cobweb plot.

Enter initial value: $q_0 := .85$ Range of plotted q-values: $q_{max} := 2.5$



Simulated time series of the real exchange rate for $time := 0 .. T_{max}$:



An equilibrium is **locally stable**, if the slope of the iterated map at this fixed point is lower than unity ($|F'(q,\mu)|<1$). Now check q_E for this:

$$F'(q,\mu) := \frac{d}{dq}F(q,\mu) \qquad \Rightarrow \qquad \overrightarrow{F'(q_E,\mu)} = \begin{bmatrix} -1.778 \\ 9.333 \\ 5.167 \end{bmatrix}$$

The slope of the iterated map can be expressed in terms of elasticities

$$F'(q,\mu) = 1 + \frac{\varepsilon(q)}{\sigma(q,\mu)} \cdot q$$

where the **trade balance elasticity** (= sum of import and export demand elasticities minus 1) is

$$\varepsilon(q) = \frac{q}{Im(q)} \cdot \frac{d}{dq}TB(q) \qquad \Leftrightarrow \qquad \varepsilon(q) := \frac{q}{(a - b \cdot q)} \cdot \left(\frac{-c}{q^2} + 2 \cdot \frac{d}{q^3} + b \right)$$

and the **elasticity of capital outflow** with respect to the expected gross yield on the foreign bonds in terms of domestic goods is

$$\sigma(q,\mu) := \frac{\mu}{(a - b \cdot q)}$$

Now we use the following proposition [Chen (1999), p. 503]:

Proposition:

Let $\sigma_{crit} := \frac{-\varepsilon(1)}{2}$, the interior equilibrium point at $q_E = 1$ undergoes a supercritical flip bifurcation when the bifurcation parameter $\sigma(1,\mu)$ passes σ_{crit}. This equilibrium is stable when $\sigma(1,\mu) > \sigma_{crit}$. It becomes unstable when $\sigma(1,\mu) < \sigma_{crit}$, and, additionally, a stable cycle of period 2 emerges.

Check for bifurcation: $\sigma(1,\mu) < \sigma_{crit} = 1$ (1 = bifurcation occurs)

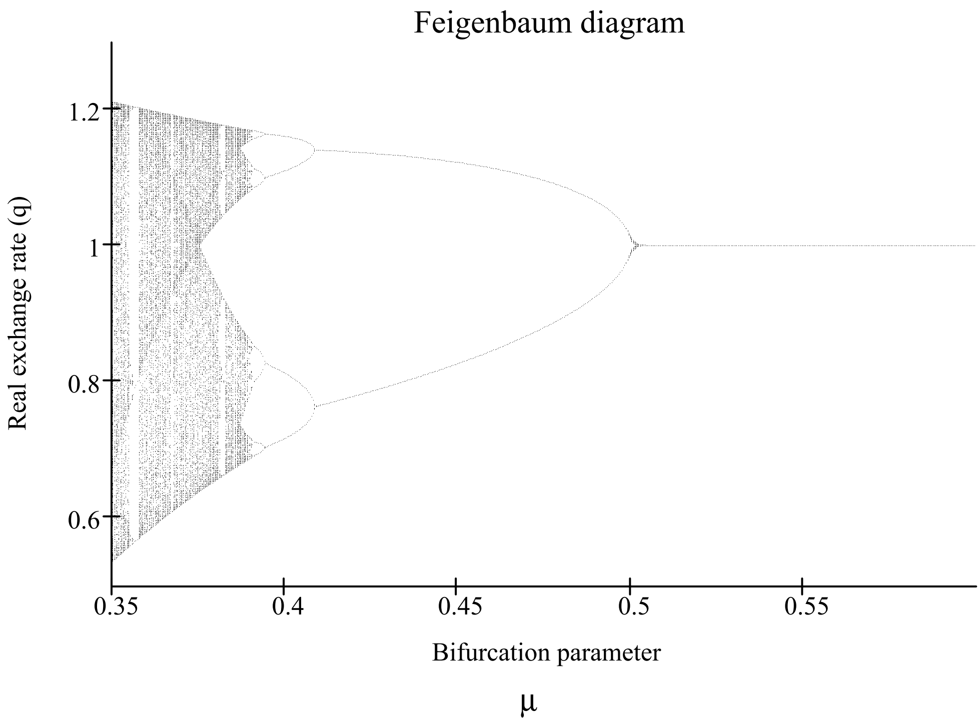
where $\sigma_{crit} = 0.1$ $\varepsilon(1) = -0.2$ $\sigma(1,\mu) = 0.072$

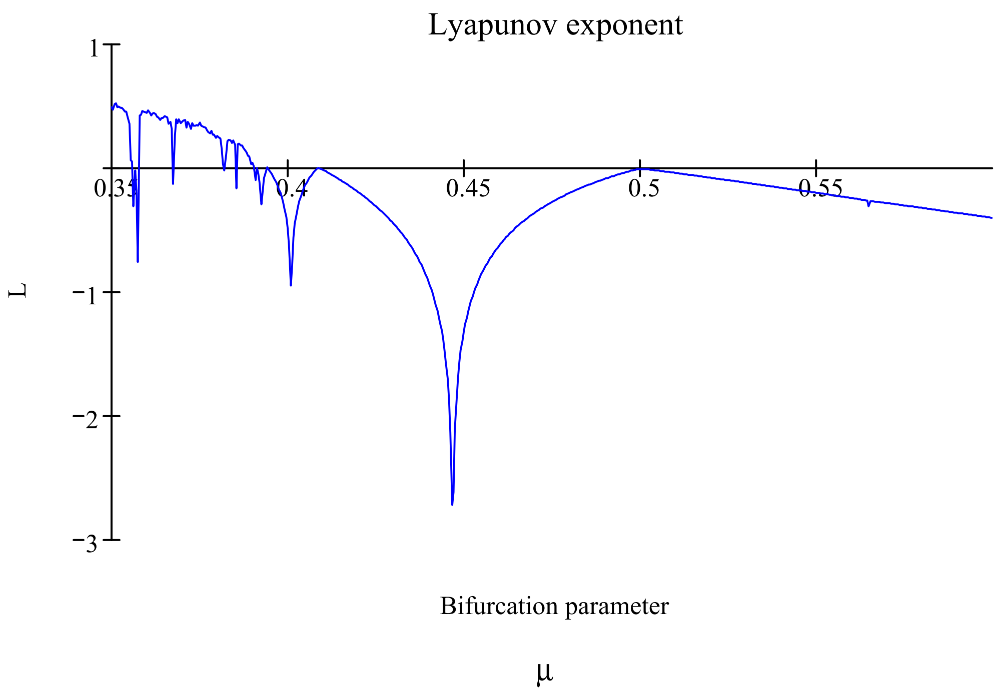
You can observe the transition to more complex dynamics of the real exchange rate by plotting the **Feigenbaum diagram** and the **Lyapunov exponent**. Use these figures to detect stable cycles of different periodicity or irregular cycles.

Resolution of graph: $RES := 6$ $(1, 2, .., 10)$

Range of plotted values: $\mu_{bottom} := 0.35$ $\mu_{top} := 0.6$

$q_{bottom} := 0.5$ $q_{top} := 1.3$





Note: Positive values of the Lyapunov exponent indicate chaotic behaviour of q!

Literature:

Chen, S.: Complex Dynamics of the Real Exchange Rate in an Open Macroeconomic Model. Journal of Macroeconomics, vol. 21 (1999), pp. 493 -508.

Sohmen, E.: Demand Elasticities and the Foreign-Exchange Market. Journal of Political Economy, vol. 65 (1957), pp. 431 - 436.